

Quiz 5**Question 1. (10 pts)**

Let $F : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation defined by

$$F(x, y) = (-3x + 4y, 4x + 3y).$$

Find the matrix representation of F with respect to the basis $S = \{u_1, u_2\} = \{(1, 2), (-2, 1)\}$.

Solution: with respect to the basis $S = \{u_1, u_2\} = \{(1, 2), (-2, 1)\}$, we have

$$[F(u_1)]_S = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

$$[F(u_2)]_S = \begin{bmatrix} 0 \\ -5 \end{bmatrix}$$

$$\text{So } [F]_S = \begin{bmatrix} 5 & 0 \\ 0 & -5 \end{bmatrix}$$

Question 2. (10 pts)

Consider the vector space $\mathbb{P}_2(t)$. Choose a basis $S = \{1, t, t^2\}$. Let T be the linear transformation defined by

$$T(f) = f'' + 4f'.$$

Find the matrix representation of T relative to the basis S .

Solution:

$$[T(1)]_S = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[T(t)]_S = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

$$[T(t^2)]_S = \begin{bmatrix} 2 \\ 8 \\ 0 \end{bmatrix}$$

So

$$[T]_S = \begin{bmatrix} 0 & 4 & 2 \\ 0 & 0 & 8 \\ 0 & 0 & 0 \end{bmatrix}$$